Goals for Today

- Lossless Lines:
  - What are the advantages when studying lossless lines?
  - What is the Surge Impedance and Surge Impedance Loading (SIL)?
  - What are the ABCD parameters and equivalent $\pi$ parameters of a lossless line?
  - What is the theoretical steady-state stability limit? And how do you compute it?
  - How does the voltage profile on a lossless line look like in different loading conditions?

- Maximum Power Flow
  - What is the theoretical steady-state stability limit under consideration of a lossy line?

- Line Loadability
  - What are line loadability limits in practice?

- Reactive Compensation Techniques
  - Why do we need reactive compensation on transmission lines?
  - What different compensation techniques are used?
**Discussion of different line models**

- Short line: Series impedance
- Medium line: nominal $\pi$ circuit
- Long lines:
  - Exact model
  - Equivalent $\pi$ circuit

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1. Lossless Lines
2. Maximum Power Flow
3. Line Loadability
4. Reactive Compensation Techniques
\( \pi \) circuit of lossless transmission

\[
\begin{align*}
I_S & \quad R \quad j \omega L \quad I_R \\
& \quad \frac{G}{2} \quad j \omega C \quad \frac{G}{2}
\end{align*}
\]

\( V_S \angle \delta \quad V_R \angle 0^\circ \)

Why such an apparently unrealistic assumption?

- Simpler expressions for line parameters
- Easy to understand concepts, such as:
  - surge impedance,
  - wavelength,
  - surge impedance loading,
  - voltage profiles and
  - steady-state stability limit.

- Transmission lines are designed to have low losses \( \Rightarrow \) Derived equations and concepts can be used for getting a quick estimate or initial design.
For a lossless line

- resistances equal to zero

\[ R = G = 0 \]

- Consequently, the impedance and the admittance are:

\[ z = j\omega L \]  
\[ y = j\omega C \]
Characteristic impedance $Z_c$

$$Z_c = \sqrt{\frac{Z}{y}}$$  \hspace{1cm} (3)

For lossless line $Z_c$:

Inserting (1) and (2) into (3), it follows:

$$Z_c = \sqrt{\frac{L}{C}} \text{ [Ω]}$$  \hspace{1cm} (4)

- commonly called *Surge Impedance* and
- purely real (resistive).

Propagation constant $\gamma$

$$\gamma = \sqrt{zy}$$  \hspace{1cm} (5)

Inserting (1) and (2) into (5), it follows:

$$\gamma = j\omega\sqrt{LC} = j\beta$$  \hspace{1cm} (6)

where

$$\beta = \omega\sqrt{LC}$$

- purely imaginary.
Lossless Lines

From last lecture, ABCD parameters for a long lossy lines

\[
A(x) = D(x) = \cosh(\gamma x) \quad (7)
\]
\[
B(x) = Z_c \sinh(\gamma x) \quad (8)
\]
\[
C(x) = \frac{\sinh(\gamma x)}{Z_c} \quad (9)
\]

For lossless transmission lines:

\[
A(x) = D(x) = \cos(\beta x) \quad (10)
\]
\[
B(x) = jZ_c \sin(\beta x) \quad (11)
\]
\[
C(x) = \frac{j \sin(\beta x)}{Z_c} \quad (12)
\]

with $\gamma = j\beta$ and

\[
\cosh(j\beta x) = \cos(\beta x)
\]
\[
\sinh(j\beta x) = j \sin(\beta x)
\]

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Resulting equation to determine voltage and current at location $x$:

$$\begin{bmatrix} V(x) \\ I(x) \end{bmatrix} = \begin{bmatrix} \cos(\beta x) & j Z_c \sin(\beta x) \\ j \frac{\sin(\beta x)}{Z_c} & \cos(\beta x) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$  \hspace{1cm} (13)

Voltage and current at the sending end:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cos(\beta l) & j Z_c \sin(\beta l) \\ j \frac{\sin(\beta l)}{Z_c} & \cos(\beta l) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$  \hspace{1cm} (14)
Lossless Lines
Equivalent \( \pi \) circuit

From last lecture, for lossy transmission line:

\[
\begin{align*}
V_S \angle \delta & \quad \begin{array}{c}
Y' \frac{Y'}{2} \quad Y' \frac{Y'}{2}
\end{array}
\quad V_R \angle 0^\circ
\end{align*}
\]

where

\[
Z' = Z \frac{\sinh(\gamma l)}{\gamma l} \quad (15)
\]

\[
Y' = \frac{Y}{2} \frac{\tanh(\gamma l/2)}{\gamma l/2} \quad (16)
\]

Lossless transmission line - nominal \( \pi \) circuit:

\[
\begin{align*}
V_S \angle \delta & \quad \begin{array}{c}
Y' \frac{Y'}{2} \quad Y' \frac{Y'}{2}
\end{array}
\quad V_R \angle 0^\circ
\end{align*}
\]

Lossless transmission line - equivalent \( \pi \) circuit parameters:

\[
\begin{align*}
Z' = Z \frac{\sinh(\gamma l)}{\gamma l} = (j\omega Ll) \frac{\sinh(j\beta l)}{j\beta l} = j\omega Ll \frac{\sin(\beta l)}{\beta l} = jX' \quad (17)
\end{align*}
\]

\[
\begin{align*}
Y' = \frac{Y}{2} \frac{\tanh(\gamma l/2)}{\gamma l/2} = \frac{j\omega Cl \tanh(j\beta l/2)}{2} \frac{\tan(j\beta l/2)}{\beta l/2} = \frac{j\omega Cl \tan(\beta l/2)}{2} \beta l/2 \quad (18)
\end{align*}
\]
Lossless Lines
Equivalent π circuit

Lossless transmission line - equivalent π circuit:

\[ Z' = j\omega l \frac{\sin (\beta l)}{\beta l} = jX' \] \hspace{1cm} (19)

\[ \frac{Y'}{2} = j\omega c l \tan \left( \frac{\beta l}{2} \right) = j\omega C' \frac{l}{2} \] \hspace{1cm} (20)

Note: \( Z' \) and \( Y' \) are both purely imaginary
\[ \Rightarrow \text{ Equivalent } \pi \text{ circuit lossless.} \]

Definition

Wavelength \( \lambda \) is the distance where the phase of the voltage or the current changed by \( 2\pi \) rad (360°).

Recalling (13)
\[ V(x) = \cos (\beta x) V_R + j \frac{Z_c}{\beta} \sin (\beta x) I_R \]
\[ I(x) = j \frac{\sin (\beta x)}{Z_c} V_R + \cos (\beta x) I_R \] \hspace{1cm} (21)

\( V(x) \) and \( I(x) \) change phase by \( 2\pi \), when \( x = \frac{2\pi}{\beta} \)

\[ \lambda = \frac{2\pi}{\beta} = \frac{1}{f\sqrt{LC}} \text{[m]} \] \hspace{1cm} (22)

with \( \beta = \omega \sqrt{LC} \) and \( \omega = 2\pi f \)
How long is the wavelength of an overhead line? Rearranging (22):

\[ f \lambda = \frac{1}{\sqrt{LC}} \left[ \frac{m}{s} \right] \]

- \( \frac{1}{\sqrt{LC}} \): Propagation velocity of voltage and current waves along a lossless line.

For overhead lines:

\[ \frac{1}{\sqrt{LC}} \approx 3 \times 10^8 \]

How long is \( \lambda \) for a overhead line operated at \( f = 50 \text{ Hz} \)?
Lossless Lines
Surge Impedance Loading (SIL)

**Definition**

Surge Impedance Loading (SIL) corresponds to line loading when lossless line is terminated with $Z_c$.

\[
\frac{V_R}{I_R} = Z_c \quad (23)
\]

\[
V_R e^{j\beta x} = V(x) = \cos (\beta x)V_R + j Z_c \sin (\beta x)I_R \quad (26)
\]

\[
I(x) = j \frac{\sin (\beta x)}{Z_c} V_R + \cos (\beta x) \frac{V_R}{Z_c} = \frac{V_R}{Z_c} e^{j\beta x} \quad (27)
\]
Lossless Lines
Surge Impedance Loading (SIL)

Apparent power $S(x)$

$$S(x) = P(x) + jQ(x) = V(x)I(x)^*$$  \hspace{1cm} (28)

With the equations for $V(x)$ and $I(x)$, compute apparent power $S(x)$

$$S(x) = V(x)I(x)^* = V_R e^{j\beta x} \left( \frac{V_R}{Z_c} e^{j\beta x} \right)^* = \frac{|V_R|^2}{Z_c}$$  \hspace{1cm} (29)

For a lossless transmission line at SIL (terminated with $Z_c$):
- real power flow $P(x)$ is constant along the line,
- reactive power flow $Q(x)$ is equal to zero and
- at rated line voltage, the real power delivered or SIL is equal to:

$$SIL = \frac{V_{\text{rated}}^2}{Z_c}$$  \hspace{1cm} (30)

- Reactive power $Q_C$ generated by capacitance $C$

$$Q_C = V_R^2 \omega C I$$  \hspace{1cm} (31)

- Reactive power $Q_L$ absorbed by inductance $L$

$$Q_L = I^2 \omega L I$$  \hspace{1cm} (32)

- At $P = SIL$: Reactive power flow $Q(x)$ is equal to zero. Consequently:

$$Q_C = Q_L \Rightarrow V_R^2 \omega C I = I^2 \omega L I$$  \hspace{1cm} (33)
Lossless Lines
Surge Impedance Loading (SIL)

- $P > SIL$, then $Q_L > Q_C$. Line has inductive behavior (absorbs reactive power).
- $P < SIL$, then $Q_C > Q_L$. Line has capacitive behavior (generates reactive power).

![Diagram showing lossless lines with surge impedance loading]

Table: Typical conductor parameters for overhead lines at 50 Hz

<table>
<thead>
<tr>
<th>Nominal voltage [kV]</th>
<th>230</th>
<th>345</th>
<th>500</th>
<th>765</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$ [Ω/km]</td>
<td>0.050</td>
<td>0.037</td>
<td>0.028</td>
<td>0.012</td>
</tr>
<tr>
<td>$\omega L$ [Ω/km]</td>
<td>0.407</td>
<td>0.306</td>
<td>0.271</td>
<td>0.274</td>
</tr>
<tr>
<td>$\omega C$ [μS/km]</td>
<td>2.764</td>
<td>3.765</td>
<td>4.333</td>
<td>4.148</td>
</tr>
</tbody>
</table>

Table: Typical conductor parameters for cables at 50 Hz

<table>
<thead>
<tr>
<th>Nominal voltage [kV]</th>
<th>115</th>
<th>230</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$ [Ω/km]</td>
<td>0.059</td>
<td>0.028</td>
<td>0.013</td>
</tr>
<tr>
<td>$\omega L$ [Ω/km]</td>
<td>0.252</td>
<td>0.282</td>
<td>0.205</td>
</tr>
<tr>
<td>$\omega C$ [μS/km]</td>
<td>192.0</td>
<td>204.7</td>
<td>80.4</td>
</tr>
</tbody>
</table>

Lossless Lines

Practical Example

**Table:** Typical conductor parameters for overhead lines at 50 Hz²

<table>
<thead>
<tr>
<th>Nominal voltage [kV]</th>
<th>230</th>
<th>345</th>
<th>500</th>
<th>765</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega L \ [\Omega/km] )</td>
<td>0.407</td>
<td>0.306</td>
<td>0.271</td>
<td>0.274</td>
</tr>
<tr>
<td>( \omega C \ [\mu S/km] )</td>
<td>2.764</td>
<td>3.765</td>
<td>4.333</td>
<td>4.148</td>
</tr>
<tr>
<td>( Z_c ) (lossless)[Ω]</td>
<td>383.7</td>
<td>285.1</td>
<td>250.1</td>
<td>257.0</td>
</tr>
<tr>
<td>( SIL ) (lossless)[MW]</td>
<td>137.9</td>
<td>417.5</td>
<td>999.6</td>
<td>2277.1</td>
</tr>
</tbody>
</table>

**Table:** Typical conductor parameters for cables at 50 Hz¹

<table>
<thead>
<tr>
<th>Nominal voltage [kV]</th>
<th>115</th>
<th>230</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega L \ [\Omega/km] )</td>
<td>0.252</td>
<td>0.282</td>
<td>0.205</td>
</tr>
<tr>
<td>( \omega C \ [\mu S/km] )</td>
<td>192.0</td>
<td>204.7</td>
<td>80.4</td>
</tr>
<tr>
<td>( Z_c ) (lossless)[Ω]</td>
<td>36.2</td>
<td>37.1</td>
<td>50.5</td>
</tr>
<tr>
<td>( SIL ) (lossless)[MW]</td>
<td>365.3</td>
<td>1425.9</td>
<td>4950.5</td>
</tr>
</tbody>
</table>


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   - Surge Impedance
   - ABCD parameters
   - Equivalent \( \pi \) circuit
   - Surge Impedance Loading
   - Voltage Profile
   - Steady-State Stability Limit

2. Maximum Power Flow

3. Line Loadability

4. Reactive Compensation Techniques
Voltage profile with fixed sending end voltage

\[ V(x) = V_s \]

\[ V(x) = V_s \]

\[ V(x) = V_s \]

\[ x = l \]

\[ x = 0 \]

\[ V_{RNL} = \frac{V_s}{\cos(\beta l)} \]

1. SIL: Voltage profile is flat
   \[ |V(x)| = |V_R| \]

2. No-load: \( I_{RNL} = 0 \) and from (13)
   \[ V_{NL}(x) = \cos(\beta x)V_{RNL} \]

3. Short circuit at receiving end: \( V_{RSC} = 0 \) and from (13)
   \[ V_{SC}(x) = Z_c \cos(\sin x)I_{RSC} \]

4. Full load: depends on \( I_{RFL} \), but lies above SC profile

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1. Lossless Lines
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   - Steady-State Stability Limit

2. Maximum Power Flow

3. Line Loadability

4. Reactive Compensation Techniques
**Definition**

*Steady-State Stability Limit* is defined by the maximum power $P_{\text{max}}$ that can be delivered through a transmission line.

How do we determine the maximum power $P_{\text{max}}$?

**Assumptions for calculation of $P_{\text{max}}$:**

- Voltage magnitudes of $V_S$ and $V_R$ are kept constant,
- Voltage angle at sending end with respect to receiving end is denoted by $\delta$

\[ Z' = jX' \quad \text{and} \quad \frac{Y'}{2} = \frac{j\omega C'I}{2} \]
The receiving end current $I_R$:

$$I_R = \frac{V_S e^{j\delta} - V_R}{jX'} - \frac{j\omega C' I}{2} V_R$$  \hspace{1cm} (34)

Complex power $S_R$ at the receiving end:

$$S_R = V_R I_R^* = V_R \left(\frac{V_S e^{j\delta} - V_R}{jX'}\right)^* + \frac{j\omega C' I}{2} V_R^2$$  \hspace{1cm} (35)

After rearrangement:

$$S_R = \frac{V_S V_R}{X'} \sin(\delta) + j \left[\frac{V_S V_R}{X'} \cos(\delta) - V_R^2 \left(\frac{1}{X'} - \frac{j\omega C'}{2}\right)\right]$$  \hspace{1cm} (36)

Active power delivered:

$$P = P_R = P_S = \frac{V_S V_R}{X'} \sin(\delta)$$  \hspace{1cm} (37)
Steady-state stability limit $P_{\text{max}}$:
- theoretical steady-state stability limit,
- if power to be delivered larger, then loss of synchronism,
- increase with the square of line voltage,
- longer lines (larger $X'$) decreases limit and
- can be expressed in pu on SIL base.

$$P_{\text{max,pu}} = \frac{P_{\text{max}}}{\text{SIL}}$$
Steady-State Stability Limit derived in terms of parameters of the equivalent π circuit

Assumptions:

- voltage magnitudes of $V_S$ and $V_R$ are kept constant,
- voltage angle at sending end with respect to receiving end is denoted by $\delta$

$$Z' = |Z'|e^{j\theta_Z} \text{ and } \frac{Y'}{2} = \frac{|Y'|}{2}e^{j\theta_Y}$$
The receiving end current $I_R$:

$$I_R = \frac{|V_S|e^{j\delta} - |V_R|}{|Z'|e^{j\theta_Z}} - \frac{|Y'|e^{j\theta_Y}}{2} V_R$$  \hspace{1cm} (38)

Complex power $S_R$ at the receiving end:

$$S_R = V_R I_R^* = |V_R| \left( \frac{|V_S|e^{j\delta} - |V_R|}{|Z'|e^{j\theta_Z}} - \frac{|Y'|e^{j\theta_Y}}{2} V_R \right)^*$$  \hspace{1cm} (39)

Rearranging $S_R$

$$S_R = \frac{|V_R||V_S|}{|Z'|} e^{j(\theta_Z - \delta)} - \frac{|V_R|^2}{|Z'|} e^{j\theta_Z} - \frac{|V_R|^2}{2} e^{-j\theta_Y}$$  \hspace{1cm} (40)

Active power $P_R$ and reactive power $Q_R$ at receiving end

$$P_R = \frac{|V_R||V_S|}{|Z'|} \cos(\theta_Z - \delta) - \frac{|V_R|^2}{|Z'|} \cos \theta_Z - \frac{|V_R|^2}{2} \frac{|Y'|}{2} \cos \theta_Y$$  \hspace{1cm} (41)

$$Q_R = \frac{|V_R||V_S|}{|Z'|} \sin(\theta_Z - \delta) - \frac{|V_R|^2}{|Z'|} \sin \theta_Z + \frac{|V_R|^2}{2} \frac{|Y'|}{2} \sin \theta_Y$$  \hspace{1cm} (42)

Theoretical maximum active power (or steady-state stability limit) reached at $\delta = \theta_Z$

$$P_R = \frac{|V_R||V_S|}{|Z'|} - \frac{|V_R|^2}{|Z'|} \cos \theta_Z - \frac{|V_R|^2}{2} \frac{|Y'|}{2} \cos \theta_Y$$  \hspace{1cm} (44)

Limit is less than for a lossless line
- $|Z'| > X'$ and
- two negative terms
Maximum Power Flow

Maximum power in terms of ABCD parameters

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   - Maximum power in terms of ABCD parameters

3 Line Loadability

4 Reactive Compensation Techniques

Complex power $S_R$ at the receiving end in ABCD parameters

$$S_R = |V_R| \left[ \frac{|V_S| e^{j \delta} - |A| |V_R| e^{j \theta_A - \theta_z}}{|Z'|} \right]^*$$  \hspace{1cm} (45)

with $A = \cosh (\gamma l) = |A| e^{j \theta_A}$ and $B = Z' = |Z'| e^{j \theta_Z}$

Active power $P_R$ at receiving end

$$P_R = \frac{|V_R||V_S|}{|Z'|} \cos (\theta_Z - \delta) - \frac{|A||V_R|^2}{|Z'|} \cos (\theta_Z - \theta_A)$$ \hspace{1cm} (46)

Reactive power $Q_R$ at receiving end

$$Q_R = \frac{|V_R||V_S|}{|Z'|} \sin (\theta_Z - \delta) - \frac{|A||V_R|^2}{|Z'|} \sin (\theta_Z - \theta_A)$$ \hspace{1cm} (47)
Remarks

in practice, transmission lines not operated at their theoretical steady-state stability limit

Practical line loadability based on:

- typical overhead line without compensation: voltage drop limit \( \frac{V_R}{V_S} \geq 0.95 \) and max. angular displacement \( (30 - 35^\circ) \)
- short lines \( (l \leq 25 \text{ km}) \): thermal rating of e.g. conductors
Reactive Compensation Techniques

Aim:
- increase line loadability
- maintain voltages near rated values

Types:
- **Shunt reactors (inductance):**
  - reduce overvoltages during light load conditions
  - reduce transient overvoltages due to switching and lightning surges
  - can reduce line loadability, if not removed during full-load conditions
- **Shunt capacitors:** support voltage during heavy load conditions
- **Static Var Compensators (SVCs):**
  - can absorb or inject reactive power
  - can minimize voltage fluctuations and increase line loadability
- **Synchronous condenser:**
  - synchronous motor with no load connected
  - can absorb or inject reactive power (slower than SVC)
- **Series capacitors:**
  - reduce line voltage drops and increase steady-state stability limit
  - disadvantage: automatic protection device needs to be installed and can introduce low-frequency oscillations (subsynchronous resonance)